International Journal of Theoretical Physics, Vol. 20, No. 5, 1981

Nonstationary Quantum Mechanics. V. Nonstationary Quantum Models of Scattering

Nickola Stefanov Todorov

Low Temperature Department of the Institute of Solid State Physics of the Bulgarian Academy of Sciences, Boulevard Lenin 72, Sofia 1184, Bulgaria

Received May 19, 1978

Some pecularities of the results of nonstationary perturbation theory in the presence of a degenerate continuous energy spectrum are considered. Their relevance to the ideology of the preceding articles in this series is discussed.

In this work we shall extend the discussion begun in (T4)[1] to the case of a continuous degenerated energy spectrum. Such a spectrum is always present when one examines a closed physical system. In most cases of this sort one is interested in the possible scatterings and reactions in the system. Momentum and energy conservation is a fundamental requirement in such cases. We shall pay special attention to energy conservation here.

In the continuous degenerated energy spectrum case one encounters some of the characteristic features of the different adiabatic models of inclusion of the particle interaction as in the discrete case. This can be seen after a relevant modification of the corresponding formulas for the different approximations of perturbation theory in the discrete case (we, certainly, examine only cases when it is applicable).

Denote by β the set of parameters completely defining a nonperturbed state, by ψ_ν the nonperturbed ("free") final state which is of interest to us and by ψ_{ν_0} the free initial state. The initial condition at $t=t_i$ is then $a_\beta^{(0)}(t_i)=\delta(\beta-\nu_0)(\psi_i=\int\delta(\beta-\nu_0)\psi_\beta\,d\beta=\psi_{\nu_0})$ and the αth correction $a_\nu^{(\alpha)}(t)$ is given by

$$a_\nu^{(\alpha)}(t)=\frac{1}{i\hbar}\int_{t_i}^{t}\int_{(\beta)}V_{\nu\beta}(t')e^{i\omega_{\nu\beta}t'}a_\beta^{(\alpha-1)}(t')\,d\beta\,dt' \tag{1}$$

[1]The notations (T1)–(T4) in the text refer, respectively, to Papers I–IV in our series "On the Nonstationary Problem in Quantum Mechanics (Todorov, 1980, 1980, 1981, 1981).

0020-7748/81/0500-0339$03.00/0

where $V_{\nu\beta}(t') = \langle \psi_\nu | V(t') | \psi_\beta \rangle$, $\omega_{\nu\beta} = -\omega_{\beta\nu} = (E_\nu - E_\beta)/\hbar$, $a_\beta(t) = d_\beta(t)\exp[i\omega_\beta t](\omega_\beta = E_\beta/\hbar)$, $d_\beta(t)$ being the actual coefficients participating in the decomposition of the perturbed wave function $\psi(t)$ over the complete set $\{\psi_\beta(x)\}$ [cf., e.g., (T4)], and $d\beta$ is a somewhat abstract but clear enough notation of integration over the said parameters.

Consider now the case when the interaction between the particles of the system is (formally) included according to the law $V(x,t) = V(x)\exp[\varepsilon t]$, $V(x)$ being the interaction operator, x denoting all the degrees of freedom of the system of interest, and $\exp[\varepsilon t](-\infty < t \leqslant 0)$, $\varepsilon = +0$, describing the specific adiabatic law of inclusion of $V(x)$ (the moment of full inclusion being $t=0$). In (T4) this law gave a transition of the system from an eigenstate of the nonperturbed Hamiltonian H_i to an eigenstate of $H_f = H_i + V(x)$. We have exactly the same result in the continuous spectrum case. Indeed, applying the general formula (1) we obtain

$$a_\nu^{(1)}(t) = \frac{V_{\nu\nu_0} e^{i\omega_{\nu\nu 0}t} e^{\varepsilon t}}{\hbar\left(\omega_{\nu_0\nu} + i\varepsilon\right)} \tag{2}$$

$$a^{(2)}(t) = \frac{e^{i\omega_{\nu\nu 0}t} e^{2\varepsilon t}}{\hbar^2\left(\omega_{\nu_0\nu} + 2i\varepsilon\right)} \int_{(\beta)} \frac{V_{\nu\beta} V_{\beta\nu_0}}{\omega_{\nu_0\beta} + i\varepsilon} d\beta \tag{3}$$

and so on. In moment $t=0$ the linear combinations with these coefficients $a_\nu^{(n)}$ will lead to an eigenfunction of H_f corresponding to the same eigenenergy as that of the initial state ψ_{ν_0} as implied by the factor $\exp[i\omega_{\nu\nu_0}t]$ in all $a_\nu^{(n)}$, $n=1,2,\ldots$ (for the case of particle scattering this is shown in most of the books on quantum scattering theory).

The transition probability density per unit time $W_{\nu\nu_0}$ corresponding to the above $a_\nu^{(n)}$, $n=1,2,\ldots$, is given by

$$W_{\nu\nu_0} = \frac{d}{dt}\left|a_\nu^{(1)} + a_\nu^{(2)} + \cdots\right|_{t=0}^2$$

$$= \frac{2\pi}{\hbar^2}\left|V_{\nu\nu_0} + \frac{1}{\hbar}\int_{(\beta)} \frac{V_{\nu\beta} V_{\beta\nu_0}}{\omega_{\nu_0\beta} + i\varepsilon} d\beta + \cdots\right|^2 \delta(\omega_{\nu\nu_0}) \tag{4}$$

in moment $t=0$ (and practically all finite t due to the factor $\exp[\varepsilon t]$, $\varepsilon \to +0$) of complete inclusion of $V(x)$. The δ-function $\delta(\omega_{\nu\nu_0})$ comes from expressions of the type

$$\lim_{\varepsilon \to +0} \frac{1}{2\pi} \frac{a\varepsilon}{(x \mp ib\varepsilon)[x \pm (a-b)i\varepsilon]} = \delta(x) \qquad (a>0, 0<b<a) \tag{5}$$

Really, when $\varepsilon \to +0$ this expression tends to zero everywhere with the exception of point $x=0$; it can be simply shown, besides, that an integration of the left-hand side of (5) over x from minus to plus infinity will give unity, so that we have all the characteristic features of a δ function. It is worth mentioning that the well-known expression

$$\lim_{\varepsilon \to +0} \frac{\varepsilon}{\pi(x^2+\varepsilon^2)} = \delta(x) \tag{6}$$

is a special case of (5) (when $a=2b$). The above δ functions guarantee energy conservation in the process.

It can be shown that, as in (T4), the first-order results for $W_{\nu\nu_0}$ of the different laws of a very slow (adiabatic) inclusion of the perturbation (interaction) coincide. This is done in the Appendix.

It is well known, however, that in the specific case of a continuous degenerated energy spectrum one can use a very different model of inclusion of the perturbation interaction $V(x)$ compared to the adiabatic ones. This model will give the same results for $W_{\nu\nu_0}$ as $V(x)\exp[\varepsilon t]$ in any order of magnitude of perturbation theory. Namely, we assume that $V(x,t)=0$, $-\infty < t < 0$, and $V(x,t)=V(x)$, $t \geqslant 0$ (a sudden inclusion of V in moment $t=0$). We have then

$$a_\nu^{(1)}(t) = \frac{V_{\nu\nu_0}}{\hbar\omega_{\nu_0\nu}}\left(e^{i\omega_{\nu_0\nu}t}-1\right) \tag{7}$$

$$a_\nu^{(2)}(t) = \frac{1}{\hbar^2}\int_{(\beta)} \frac{V_{\nu\beta}V_{\beta\nu_0}}{\omega_{\nu_0\beta}+i\varepsilon}\left(\frac{e^{i\omega_{\nu\nu_0}t}-1}{\omega_{\nu_0\nu}} - \frac{e^{i\omega_{\nu\beta}t}-1}{\omega_{\beta\nu}}\right)d\beta$$

$$= \frac{1}{\hbar^2}\int_{(\beta)} \frac{V_{\nu\beta}V_{\beta\nu_0}}{\omega_{\nu_0\beta}+i\varepsilon}\,\frac{e^{i\omega_{\nu\nu_0}t}-1}{\omega_{\nu_0\nu}}\,d\beta \tag{8}$$

where $\varepsilon = +0$; the appearing of $i\varepsilon$ in the denominator of the second equality (8) is explained, e.g., by Feinman and Hibbs (1965) [but one must bear in mind that in their expression (6.100), e.g., the second term is given with an erroneous sign]. The presence of a factor $(e^{i\omega_{\nu\nu_0}t}-1)/\omega_{\nu_0\nu}$ will lead to the uncertainty relation $\Delta E \Delta t \sim \hbar$ in the expression for $W_{\nu\nu_0}$ given below. It is a straightforward consequence of the well-known formula

$$\lim_{t\to\infty} \frac{\sin xt}{\pi x} = \delta(x) \tag{9}$$

the application of which gives

$$W_{\nu\nu_0} = \lim_{t\to\infty} \frac{d}{dt}\left|a_\nu^{(1)}(t) + a_\nu^{(2)}(t) + \cdots\right|^2$$

$$= \frac{2\pi}{\hbar^2}\left|V_{\nu\nu_0} + \frac{1}{\hbar}\int_{(\beta)} \frac{V_{\nu\beta}V_{\beta\nu_0}}{\omega_{\nu_0\beta} + i\varepsilon} d\beta + \cdots\right|^2 \delta(\omega_{\nu\nu_0}) \tag{10}$$

[using the argumentation of Feynman and Hibbs (1965) one can see that (10) will coincide with (4) in any order of magnitude]. The δ function in (10) plays the same role as the one in (4).

The coincidence of the total expressions (4) and (10) has a clear physical sense. After a transitory period of time determined by the relation $\Delta E \Delta t \sim \hbar$ the perturbing effects of the sudden inclusion of $V(x)$ on the initial state of motion become negligible compared to the net effect of the potential for large time intervals and we gradually come to the quantities determined by $V(x)$ only. In the case $V(x)\exp[\varepsilon t]$ we have always the net effect of $V(x)$ only due to the smallness of ε. The specific way of action leading to the δ functions $\delta(\omega_{\nu\nu_0})$ in the expressions for the different $W_{\nu\nu_0}$ shows that we are really dealing with a scattering-and-reactions problem in which several initially noninteracting parts of a closed overall system ($\psi_i = \psi_{\nu_0}$) come into contact with each other and, after interaction, some reaction products are scattered in far-off space regions where again no interaction exists and one may use, as before scattering, some "free" eigenfunctions of the energy operator ($\psi_f = \psi_\nu$). The conservation of energy in all such processes, guaranteed by $\delta(\omega_{\nu\nu_0})$ on the other hand, shows that the problem can be solved in a static variant of a fixed eigenenergy too. The results for the transition rates given by the nonstatic and the static variants of scattering coincide. Having in mind the above discussion of the decreasing importance of initial perturbing effects we see that the said properties of $V(x)\exp[\varepsilon t]$ and the "instantaneous" potential show that coincidence with the static results has to be expected in an arbitrary process of a very slow inclusion of a potential $V(x, t)$. This potential will be practically equal to $V(x)$ in a sufficiently large time interval, so that we shall have the net effect about $V(x)$ *in this interval*. This is demonstrated in the Appendix with the three cases examined there in the first order of nonstationary perturbation theory.

It is relevant to return here to the postulates (1) and (2) in (T1) of present-day quantum theory. We shall denote them further as (A) and (B), correspondingly. It was argued in (T1) that (A) can lead to difficulties of both logics and specific numbers in some cases when one can formally use it. Basic requirements neglected by (A) such as conservation laws are taken

into consideration in (B) (T1). But there immediately appears a problem: How can one use (B) in the general case? The discussion of the nonstationary models of scattering in the present Paper V shows that the results of quite different models coincide with those of the stationary method. This fact is important since it clearly demonstrates the significance of conservation laws in quantum mechanics. It implies that, most probably, (B) will give reliable results in such cases of a presence of a time-dependent interaction in different physical situations, in which conservation laws in the overall system are taken into consideration (e.g., energy conservation in a closed system consisting of atoms and photons). This may explain why the rule $2\pi|V_{\nu\nu_0}|^2\rho(E_\nu)/\hbar$ of calculation of transition rates is really "golden," as Fermi called it; we have $E_\nu = E_{\nu_0}$ in this formula and energy conservation is taken into account in the corresponding integration

$$\frac{2\pi}{\hbar}\int |V_{\nu\nu_0}|^2\,\delta(E_\nu - E_{\nu_0})\rho(E_\nu)\,dE_\nu$$

where $\rho(E_\nu)$, certainly, is a relevant density-of-states function.

The Schrödinger equation (SE) for a part of an overall system, even when formally applicable [well-defined externally in respect to the subsystem field $U(t)$] cannot be relied on, generally, about the specific numerical probability values, contrary to the wide-spread belief that it can. The said common opinion is based on the concept that in a relevant process of averaging, the parameters of the (large) part of the overall system with which the subsystem of interest interacts disappear entirely from the problem and what remains is calculation of probability values with the help of (A) for our subsystem in the field $U(t)$ without the necessity of any additional information. But a concept needs a proof in physics and one of the aims of our series of articles on the nonstationary problem in quantum mechanics was to demonstrate that some facts do not agree with the said opinion. Namely, in (T3) we came to disagreement with classical mechanics in the case of a quasiclassical motion, while in (T4) the application of the nonstationary SE to a part of a system led to the appearing of some transition probabilities which we qualified as nonphysical in the cases of an adiabatic (i.e., very slow but otherwise arbitrary) variation of the external field. This shows, at least, that the information only, necessary according to (A) for the complete determination of probability values, is in fact insufficient in the general case. The same applies to that part of the discussion in (T2) in which we showed that (A) cannot ensure an increase with time of the average value of the entropy operator $\hat{S}$ defined there. Indeed, if one accepts the transition probability values given by (A) he shall always have $\langle\hat{S}\rangle_{t=0} = \langle\hat{S}\rangle_{t>0}$ for arbitrary t ($t=0$ in the moment of a sudden increase of the volume in which our system is held). The increase

of $\langle \hat{S} \rangle$ with time is obtained with the help of the natural requirement that the final state to which our many-body system strives in the process of its irreversible evolution in the specific case examined is of (practically) the same energy as the initial one (in the initial volume), contrary to what the set of probabilities obtained with the help of (A) gives. And last (but not least) in Appendix B of (T1) we discussed a situation in which a difficulty of logics exists if one relies only on the initial information necessary according to (A) for the complete solution of a problem in the case of a formal applicability of this postulate. Some additional information about the possible position of the particle in the moment of appearing of the perturbation is necessary in order to avoid the difficulty. Cases of this sort are very complex and one cannot be sure a priori that postulate (B) will be able to describe situations with such a precise and complicated conduct of the perturbing field. But the inference about the necessity of some additional information (hidden variables) for the description of the conduct of simpler subsystems implied by the reinterpretation of the solutions of (B) given in (T1) is general and fully applies to the particle in the potential well described in the said Appendix.

Some experimental facts discussed in (T1) seem to give evidence too about a strong inapplicability of (A) for the calculation of transition rates in the specific cases considered there. However, careful additional work is still necessary, in our opinion, in order to draw a precise picture of the actual state of affairs in the said field, as was pointed out in (T1).

The problem of time irreversibility discussed extensively in different papers of our series is connected with the behavior of a subsystem of a larger system in the process of its isolation from the remaining part. It is evident that in cases of quasi-instantaneous perturbations the theory of nonstationary perturbations, applied to the overall system, provides a correct qualitative picture too of the way in which all the noninteracting subsystems of the said system reach states of fixed energies ($\Delta E \Delta t \sim \hbar$—see the discussion on this above; the way of application of perturbation theory leading to this relation is common for arbitrary systems). This picture should be valid in any case when one can discern physically a specific moment in the evolution of a closed system without changing with this its energy ($E_f = E_i$) even when the perturbation is not weak, since perturbation theory, clearly, gives evidence about a general phenomenon. The existence of this phenomenon exactly, combined with the lack of mutual influence postulated by conventional quantum mechanics itself [in the sense discussed in (T1)] of the conduct of noninteracting subsystems together with the requirement that a state of motion should be an objective characteristics of a system, equal for all observers, was employed in (T1), (T2), and (T4) in order to show that the SE does not give, generally, acceptable results about

evolution in time when applied to a part of an overall system. Indeed, the moduli of the coefficients of the corresponding wave packets in (T1) and (T2), representing the states of subsystems described by the SE in the case of variable external potentials $U(t)$, do not depend on time in the specific cases of interest there, in contradiction with the physical sense of $\Delta E \Delta t \sim \hbar$ considered in the present paper and the above-mentioned requirements. On the other hand, the nonphysical terms resulting from the application of (A) to the cases discussed in (T4) are removed (on a heuristic basis) with the help of the concept about fundamental irreversibility of the basic evolution equation(s). Inherent time irreversibility in the conduct of any system (since any system can interact with other ones) with the exception of systems which have been isolated for a sufficiently large interval of time and have reached their "equilibrium" eigenstates (fixed E) is thus an important concept in our series of articles on the nonstationary problem in quantum mechanics. This concept clearly indicates that present-day quantum theory is not a complete theory in accord with the Einstein–Podolsky–Rosen (1935) theorem. Namely, neither (A) nor (B) gives information about the specific way in which the concrete noninstantaneous irreversible reduction of some initial wave packet of a given subsystem to a fixed stationary state takes place. Indeed, (B) gives only some (correct) overall picture of all the possibilities and some qualitative information about the way in which the total system will come to a state of definite energy (the above uncertainty relation). But, as was emphasized in (T1), if something exists objectively (relaxation processes in subsystems), it certainly needs an objective description.

ACKNOWLEDGMENTS

We are greatly indebted to Dr. E. Leyarovski for his tolerance and friendly goodwill, which made the completion of this series of articles possible. Thanks are due, besides, to Dr. M. Bushev and D. Uzunov for valuable discussions on the material of Paper V and to the referee for constructive remarks which led to improvements in the exposition of Papers I and II of the series.

APPENDIX

We shall examine here the three specific adiabatic laws of inclusion of a perturbation $V(x, t)$, x denoting all the degrees of freedom of the system, considered in detail in (T4) in the case of a discrete energy spectrum. They are

(a) $V(x,t)=0, \quad t<0; \quad V(x,t)=tV(x)/T, \quad 0 \leqslant t \leqslant T, \quad T \to \infty$

(b) $V(x,t)=0, \quad t<0; \quad V(x,t)=t^2V(x)/T^2, \quad 0\leqslant t\leqslant T, \quad T\to\infty$

(c) $V(x,t)=0, \quad t<0; \quad V(x,t)=V(x)\sin t, \quad 0\leqslant t\leqslant T, \quad T\to\infty$

$\omega=\pi/2T.$

In the case (a) we have

$$a_\nu^{(1)}(t)=\frac{V_{\nu\nu_0}}{\hbar T\omega_{\nu_0\nu}}\left[te^{i\omega_{\nu\nu_0}t}-\frac{1}{i\omega_{\nu\nu_0}}(e^{i\omega_{\nu\nu_0}t}-1)\right] \tag{A.1}$$

$$\lim_{T\to\infty}\frac{d}{dt}|a_\nu^{(1)}(t)|^2_{t=T}=\lim_{T\to\infty}\frac{|V_{\nu\nu_0}|^2 2t}{\hbar^2T^2\omega_{\nu_0\nu}^2}(1-\cos\omega_{\nu\nu_0}t)_{t=T}$$

$$=\frac{2\pi}{\hbar^2}|V_{\nu\nu_0}|^2\,\delta(\omega_{\nu\nu_0}) \tag{A.2}$$

where the well-known expression

$$\lim_{T\to\infty}\frac{\sin^2 xT}{\pi Tx^2}=\delta(x) \tag{A.3}$$

is applied. Equation (A.2) coincides with the first-order result for $V(x,t)=V(x)\exp[\varepsilon t]$.

In the case (b) one obtains

$$a_\nu^{(1)}(t)=\frac{V_{\nu\nu_0}}{\hbar T^2\omega_{\nu_0\nu}}\left[\left(t^2-\frac{2t}{i\omega_{\nu\nu_0}}\right)e^{i\omega_{\nu\nu_0}t}-\frac{2}{\omega_{\nu\nu_0}^2}(e^{i\omega_{\nu\nu_0}t}-1)\right] \tag{A.4}$$

$$\lim_{T\to\infty}\frac{d}{dt}|a_\nu^{(1)}(t)|^2_{t=T}=\lim_{T\to\infty}\frac{4|V_{\nu\nu_0}|^2}{\hbar^2T\omega_{\nu_0\nu}^2}\left(1-\frac{\sin\omega_{\nu\nu_0}T}{\omega_{\nu\nu_0}T}\right)$$

$$=\lim_{T\to\infty}\frac{4|V_{\nu\nu_0}|^2}{\hbar^2T\omega_{\nu_0\nu}^2}\left(1-\cos\omega_{\nu\nu_0}T+\cos\omega_{\nu\nu_0}T-\frac{\sin\omega_{\nu\nu_0}T}{\omega_{\nu\nu_0}T}\right) \tag{A.5}$$

The sum of the first two terms in the brackets gives, evidently, $4\pi|V_{\nu\nu_0}|^2\delta(\omega_{\nu\nu_0}/)\hbar^2$ when multiplied by $4|V_{\nu\nu_0}|^2/\hbar^2T\omega_{\nu_0\nu}^2$, $T\to\infty$ [see equa-

tion (A.2)]. When $T\to\infty$ the term

$$\frac{T}{T^2\omega_{\nu\nu_0}^2}\left(\cos\omega_{\nu\nu_0}T-\frac{\sin\omega_{\nu\nu_0}T}{\omega_{\nu\nu_0}T}\right)$$

tends to zero for every $\omega_{\nu\nu_0}\neq 0$ and the integral

$$\int_{-\infty}^{\infty}\frac{T}{T^2\omega_{\nu\nu_0}^2}\left(\cos\omega_{\nu\nu_0}T-\frac{\sin\omega_{\nu\nu_0}T}{\omega_{\nu\nu_0}T}\right)d\omega_{\nu\nu 0}=2\int_0^{\infty}\frac{1}{x}d\frac{(\sin x)}{x}=-\frac{\pi}{2} \quad (A.6)$$

($x=\omega_{\nu\nu_0}T$). This shows that

$$\lim_{T\to\infty}\frac{2}{\pi T\omega_{\nu\nu_0}^2}\left(\frac{\sin\omega_{\nu\nu_0}T}{\omega_{\nu\nu_0}T}-\cos\omega_{\nu\nu_0}T\right)=\delta(\omega_{\nu\nu_0}) \quad (A.7)$$

is a representation of the δ function. Combining the above expressions we come once again to the result (A.2), this time for the t^2/T^2 law.

In the case (c) one obtains

$$a_\nu^{(1)}(t)=\frac{iV_{\nu\nu_0}}{2\hbar}\left(\frac{e^{i(\omega_{\nu\nu 0}+\omega)t}-1}{\omega_{\nu\nu_0}+\omega}-\frac{e^{i(\omega_{\nu\nu 0}-\omega)t}-1}{\omega_{\nu\nu_0}-\omega}\right) \quad (A.8)$$

$$\frac{d}{dt}|a_\nu^{(1)}(t)|_{t=T}^2=\frac{|V_{\nu\nu_0}|^2}{\hbar^2}\left(\frac{1}{\omega_{\nu\nu_0}+\omega}-\frac{1}{\omega_{\nu\nu_0}-\omega}\right)\cos\omega_{\nu\nu_0}T \quad (A.9)$$

Because of the fast oscillations of $\cos\omega_{\nu\nu_0}T$ with the variation of $\omega_{\nu\nu_0}$ when $T\to\infty$, only the region $\omega_{\nu\nu_0}\approx 0$ is of interest to us. Integration over $\omega_{\nu\nu_0}$ gives

$$\int_{-\infty}^{\infty}\left(\frac{1}{\omega_{\nu\nu_0}+\omega}-\frac{1}{\omega_{\nu\nu_0}-\omega}\right)\cos\left(\frac{\pi\omega_{\nu\nu_0}}{2\omega}\right)d\omega_{\nu\nu_0}$$

$$=\pi\int_0^{\infty}\sin\left(\frac{\pi x}{2}\right)\ln\left|\frac{x+1}{x-1}\right|dx=2\pi \quad (A.10)$$

where we have used the formula 4.382.1 (p. 597)

$$\int_0^{\infty}\ln\left|\frac{x+a}{x-a}\right|\sin bx\,dx=\frac{\pi}{b}\sin ab \qquad (a>0, b>0) \quad (A.11)$$

In the manual of Gradshtein and Ryzik (1971). Thus we come once again to the formula $2\pi|V_{\nu\nu_0}|^2\,\delta(\omega_{\nu\nu_0})/\hbar^2$, the expression

$$\lim_{\omega\to 0}\frac{\omega}{\omega^2-\omega_{\nu\nu_0}^2}\cos\left(\frac{\pi\omega_{\nu\nu_0}}{2\omega}\right)=\delta(\omega_{\nu\nu_0}) \tag{A.12}$$

being a representation of the δ function.

As the reader has most probably noticed, the terms on the right-hand side of equations (A.1), (A.4), and (A.8) which do not contain a factor of the type $\exp[i\omega_{\nu\nu_0}t]$ play an essential role in ensuring the coincidence of the first-order expressions $a_\nu^{(1)}(t)$ in these cases with that for the case $V(x)\exp[\varepsilon t]$. The removal of similar terms, necessary in the discussion in (T4), is inadmissible here since they are used in fact for the calculation of the limits of

$$\frac{d}{dt}|a_\nu^{(1)}(t)|^2_{t=T} \qquad \text{for} \quad \omega_{\nu\nu_0}\to 0$$

(being the only ones $\neq 0$ when $T\to\infty$) in which cases $\exp[i\omega_{\nu\nu_0}t]=\text{const}=1$ in all the said terms. In such a way when one is interested in the values of $a_\nu^{(1)}(t)$ for $E_\nu=E_{\nu_0}$ one cannot say that the terms of the type $\exp[i\omega_{\nu\nu_0}t]$ ($\omega_{\nu\nu_0}=0$) are of a different nature compared to terms that do not contain such a factor, and all possible terms must be taken into consideration. The specific character and purpose of our way of proceeding in the case when an isolated system is of interest (presence of a continuous spectrum) makes thus unneccessary and impossible any renormalization procedure in accord with the idea in the present paper that nonstationary perturbation theory will lead to physically sensible results when applied to such a system. Let us recall that the nonphyiscal terms in (T4) appeared in the case of a system in a varying *external* field when no mechanical conservation laws can be required for the subsystem formed by the said system in the time-dependent field.

REFERENCES

Einstein, A., Podolsky, B., and Rosen, N. (1935). *Physical Review*, **47**, 777.

Feynman, R. P., and Hibbs, A. R. (1965). *Quantum Mechanics and Path Integrals*, McGraw-Hill, New York, Chap. 6.

Gradshtein, I. S., and Ryzik, I. M. (1971). *Tables of Integrals, Sums, Series and Products*, Nauka, Moscow.

Todorov, N. S. (1980). *International Journal of Theoretical Physics*, **19**, 605.

Todorov, N. S. (1980). *International Journal of Theoretical Physics*, **19**, 725.

Todorov, N. S. (1981). *International Journal of Theoretical Physics*, **20**, 51.

Todorov, N. S. (1981). *International Journal of Theoretical Physics*, in press.